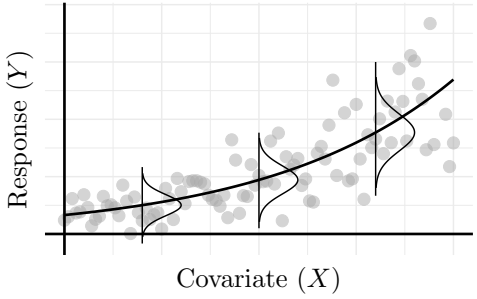
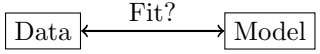
Bootstrap-Based Goodness-of-Fit Test for Distributional Regression

Distributional regression



Goodness-of-fit



$$H_0 : F_{Y|X} \in \mathcal{F} = \{(x, y) \mapsto F_\vartheta(y|x)\}_{\vartheta \in \Theta}$$

Test statistic

$$\|\tilde{\alpha}_n\| = \sup_{t \in \mathbb{R}} \left| n^{-1/2} \sum_{i=1}^{n} I_{\{Y_i \le t\}} - F_{\hat{\vartheta}_n}(t|X_i) \right|$$

Weak convergence: $\tilde{\alpha}_n \Rightarrow \tilde{\alpha}_\infty$ under H_0

Bootstrap method

$$X_i^* = X_i, \quad Y_i^* \sim F_{\hat{\vartheta}_n}(\cdot|X_i^*)$$

Weak convergence: $\tilde{\alpha}_n^* \Rightarrow \tilde{\alpha}_\infty$

Consistency

$$\mathbb{P}_{H_1}(\|\tilde{\alpha}_n\| > c_\alpha) \xrightarrow[n \to \infty]{} 1$$

Simulation studies

Rejection rate at 5% significance:

Method	H_0	H_1
New approach	5.8%	48.6%
Andrews (1997)	5.2%	9.8%
Bierens & Wang (2013)	4.7%	4.1%
Dikta & Scheer (2021)	4.6%	0.0%

$\longrightarrow$ Empirical evidence of higher power

Implementation

"gofreg"-package on CRAN